5.8 $b_k = \{13, -13, 13\}$

$$x[n] = \begin{cases} 0 & \text{for } n \text{ even} \\ 1 & \text{for } n \text{ odd} \end{cases}$$

$$y[n] = h[n] * x[n] = ?$$

1. $X[n] * \delta[n] = x[n]$
2. $X[n] * \delta[n-k] = x[n-k]$

$\delta[n] * x(n)$ $\quad h[n] = 13\delta[n] - 13\delta[n-1] + 13\delta[n-2]$

$$\begin{array}{c|cccccccc}
 & & & & \uparrow 0 & & & & \\
\delta[n] * x(n) & 13 & 0 & 13 & 0 & 13 & 0 & 13 & 0 \\
\delta(n-1) & 0 & -13 & 0 & -13 & 0 & -13 & 0 & -13 \\
\delta(n-2) & 13 & 0 & 13 & 0 & 13 & 0 & 13 & 0 \\
\hline
 & 26 & -13 & 26 & -13 & 26 & -13 & 26 & -13
\end{array}$$

$$y[n] = \begin{cases} -13 & \text{for } n \text{ even} \\ 26 & \text{for } n \text{ odd} \end{cases}$$

6.1 $x[n] = e^{j(0.4\pi n - 0.5\pi)}$

$y[n] = x[n] - x[n-1]$

Is it possible to write $y[n] = Ae^{j(\hat{\omega}_0 n + \phi)}$

$= e^{-j\pi/2} \cdot e^{j0.4\pi n} - e^{-j\pi/2} \cdot e^{j0.4\pi(n-1)}$

$= e^{-j\pi/2} \cdot e^{j0.4\pi n} \underbrace{\left(1 - e^{-j0.4\pi}\right)}$

$1 - a - bj$

$(1-a) - bj$

$\sqrt{(1-a)^2 + b^2} = A$

$\phi \leftarrow \tan^{-1}\left(\frac{-b}{1-a}\right) = \theta$ $\quad \underline{1.17 e^{j0.3\pi}}$

$= \underbrace{1.17}_{A} \cdot e^{j\underbrace{(-0.2)\pi}} \cdot e^{j\underbrace{0.4\pi}_{\hat{\omega}} n}$

6.2 $y[n] = (x[n])^2$

when $x[n] = A \cdot e^{j\phi} \cdot e^{j\hat{\omega} n}$

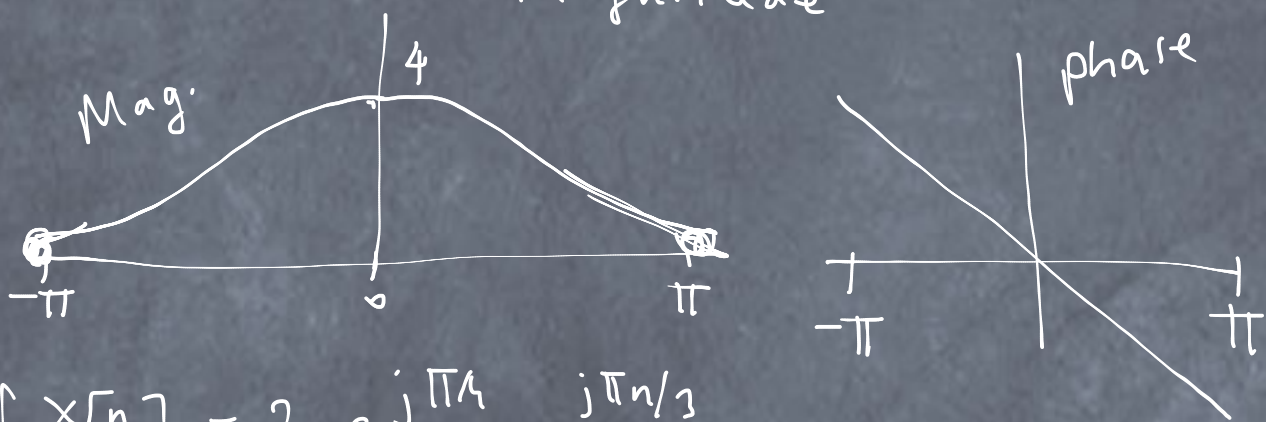
$y = A^2 \cdot e^{j2\phi} \cdot e^{j2\hat{\omega} n}$ → since the function is not linear, frequency has changed to $2\hat{\omega}$

Ex: $y[n] = x[n] + 2x[n-1] + x[n-2]$

$H(e^{j\omega}) = 1 + 2 \cdot e^{-j\omega} + e^{-j\omega 2}$

$= e^{-j\omega}(e^{j\omega} + 2 + e^{-j\omega})$

$= \underbrace{e^{-j\omega}}_{\text{phase}} \underbrace{(2\cos\omega + 2)}_{\text{Magnitude}}$




If $x[n] = 2 \cdot e^{j\pi/4} \cdot e^{j\pi n/3}$

$|H(e^{j\pi/3})| = 3$ Phase: $-\omega \rightarrow -\frac{\pi}{3}$

$y[n] = 2.3 \cdot e^{j\pi/4} \cdot e^{-j\pi/3} \cdot e^{j\pi n/3}$

6.4 $y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$

a. Find freq. resp. $H(e^{jw})$ with mag. & phase components.

$$H(e^{jw}) = 2 - 3.e^{-jw} + 2.e^{-jw2}$$

$$= e^{-jw}(2.e^{jw} - 3 + 2.e^{-jw})$$

$$= \underbrace{e^{-jw}}_{\text{phase}}\underbrace{(4\cos w - 3)}_{\text{mag.}\ (-7,1)}$$

6.5 $e^{-jw}(2+2\cos w) = H(e^{jw})$

ⓒ When input $10+4\cos(0.5\pi n+\pi/4)$

$H(0) = 4e^{j0} = 4$

$H(\frac{\pi}{2}) = 2 \cdot e^{-j\pi/2}$

$H(-\pi/2) = 2 \cdot e^{j\pi/2}$

$y[n] = 10 \cdot H(0) + H(\pi/2) \cdot 2 \cdot e^{j\pi/4} \cdot e^{j\frac{\pi}{2}n} + 2 \cdot H(-\pi/2) \cdot e^{-j\pi/4} \cdot e^{-j\frac{\pi}{2}n}$

$y[n] = 40 + 2 \cdot e^{-j\pi/2} \cdot 2 \cdot e^{j\pi/4} \cdot e^{j\frac{\pi}{2}n} + 2 \cdot e^{j\pi/2} \cdot e^{j-\pi/4} \cdot e^{-j\frac{\pi}{2}n}$

$= 40 + 8\cos(\frac{\pi}{2}n - \frac{\pi}{4})$

$2 + 2\cos w = 0$

$\cos w = -1$

$w = \pi, -\pi$

6.6 $y[n] = x[n] - x[n-2]$

Freq. resp., mag. & phase.

$H(w) = 1 - e^{-j2w}$

$= e^{-jw}\left(\frac{e^{jw} - e^{-jw}}{2j}\right) \cdot 2j$

$= e^{-jw}(2\sin w) \cdot e^{j\pi/2}$

(c) $x[n] = 4 + \cos(\frac{\pi}{4}n - \pi/4)$

$H(0)$   $H(\pi/4)$

$H(0) = 0$

$H(\pi/4) = \sqrt{2}\, e^{j\pi/4}$   $y[n] = \sqrt{2}\cos(\frac{\pi}{4}n - \frac{\pi}{4} + \frac{\pi}{4})$

$H(-\pi/4) = -\sqrt{2}\, e^{j3\pi/4}$   $= \sqrt{2}\cos(\frac{\pi}{4}n)$

$4 + \frac{1}{2} \cdot e^{j\frac{\pi}{4}n} \cdot e^{-j\frac{\pi}{4}} - \frac{1}{2} \cdot e^{-j\frac{\pi}{4}n} \cdot e^{j\pi/4}$

$H(0)$   $H(\frac{\pi}{4})$   $H(-\frac{\pi}{4})$

$= \frac{\sqrt{2}}{2} \cdot \cancel{e^{j\pi/4}} \cdot e^{j\frac{\pi}{4}n} \cdot \cancel{e^{-j\pi/4}} - \frac{\sqrt{2}}{2} \cdot e^{-j\pi/4 n} \cdot e^{j\pi}$

$= \frac{\sqrt{2}}{2} \cdot \left(e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}\right)$

$= \sqrt{2}\cos(\frac{\pi}{4}n)$

6.7 $H(e^{j\omega}) = 1 + 2 \cdot e^{-j3\omega}$

$h[n] = \delta[n] + 2\delta[n-3]$

(b) $H(e^{j\omega}) = 2 \cdot e^{-j3\omega} \cos\omega$

$= e^{-j3\omega}(e^{j\omega} + e^{-j\omega})$

$= e^{-j2\omega} + e^{-j4\omega}$

$h[n] = \delta[n-2] + \delta[n-4]$

6.8 $H(e^{j\omega}) = (1+e^{-j\omega})(1-e^{j\frac{2\pi}{3}} \cdot e^{-j\omega})(1-e^{-j\frac{2\pi}{3}} e^{-j\omega})$

a. Diffence Eqn.

$= (1+e^{-j\omega})(1-e^{-j\frac{2\pi}{3}} \cdot e^{-j\omega} - e^{j\frac{2\pi}{3}} e^{-j\omega} + e^{-j2\omega})$

$= (1+e^{-j\omega})(1-e^{-j\omega} + e^{-j2\omega})$

$= 1 + e^{-j3\omega}$ $\qquad y[n] = x[n] + x[n-3]$

b. Imp. Resp. $h[n] = \delta[n] - \delta[n-3]$

6.14 $y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$

a. $h[n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3]$

$= \frac{1}{4}(1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega})$

$= \frac{1}{4} \cdot e^{-j\frac{3}{2}\omega}\left(e^{j\frac{3}{2}\omega} + e^{j\frac{1}{2}\omega} + e^{-j\frac{1}{2}\omega} + e^{-j\frac{3}{2}\omega}\right)$

$= \frac{1}{2} e^{-j\frac{3}{2}\omega}\left(\cos\left(\frac{3}{2}\omega\right) + \cos\left(\frac{1}{2}\omega\right)\right)$

d. If input $x[n] = 5 + 4\cos(0.2\pi n) + 3\cos(0.5\pi n)$

$H(0) = 1$

$H(0.2\pi) = 0.7e^{-j0.3\pi}$

$H(0.5\pi) = 0$

$y[n] = 5 + 4 \cdot (0.7) \cdot \cos(0.2\pi n - 0.3\pi) + 0$