CMPE 350 - Spring 2015
PS Questions

PS 1 - 16.02.15

1.6 Give state diagrams of DFAs recognizing the following languages. In all parts the alphabet is $\{0, 1\}$.

a) $\{w|w$ begins with a 1 and ends with a 0$\}$

d) $\{w|w$ has length at least 3 and its third symbol is a 0$\}$

f) $\{w|w$ doesn't contain the substring 110$\}$

h) $\{w|w$ is any string except 11 and 111$\}$

i) $\{w|$every odd position of w is a 1$\}$

1.36 Let $B_n = \{a^k|$ where k is a multiple of $n\}$. Show that for each $n > 1$, the language B_n, is regular.

• x is a prefix of string y if a string z exists where $xz = y$. Let A be a regular language and let $L_A = \{x|\exists$ a string z such that $xz \in A\}$. Prove that L_A is regular.

• If a DFA with n states accepts a string of length $n-1$, then it also accepts infinitely many other strings.

PS 2 - 23.02.15

1.7 Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts the alphabet is $\{0, 1\}$.

b) $\{w|w$ contains the substring 0101 i.e $w = x0101y$ for some x and $y\}$

c) $\{w|w$ contains an even number of 0s or contains exactly two 1s$\}$

1.14 a) Show that if M is a DFA that recognizes language B, swapping the accept and nonaccept states in M yields a new DFA recognizing the complement of B. Conclude that the class of regular languages is closed under complement.

b) Show by giving an example that if M is an NFA that recognizes language C, swapping the accept and nonaccept states in M doesn't necessarily yield a new NFA that recognizes the complement of C. Is the class of languages recognized by NFAs closed under complement? Explain your answer.

1.31 For any string $w_1 w_2 \ldots w_n$ the reverse of w, written w^R, is the string w in reverse order, $w_n \ldots w_2 w_1$. For any language A, let $A^R = \{w^R | w \in A\}$. Show that if A is regular, so is A^R.

1.43 Let A be any language. Define DROP-OUT(A) to be the language containing all strings that can be obtained by removing one symbol from a string in A. Thus, DROP-OUT(A) $=$ $\{xz | xyz \in A$ where $x, z \in \Sigma^*, y \in \Sigma\}$. Show that the class of regular languages is closed under the DROP-OUT operation. Give both a proof by picture and a more formal proof by construction as in Theorem 1.47.

• If a language A is finite then it is regular. Show that the converse is not always true.

PS 3 - 02.03.15

1.29 Use the pumping lemma to show that the following languages are not regular.

b) $A_2 = \{www | w \in \{a,b\}^*\}$

c) $A_3 = \{a^{2^n} | n \geq 0\}$

1.46 Prove that the following regular languages are not regular. You may use the pumping lemma and the closure properties of the class of regular languages under union, intersection and complement.

a) $L = \{0^n 1^m 0^n | m, n \geq 0\}$

b) $L = \{0^m 1^n | m \neq n\}$

c) $L = \{w | w \in \{0,1\}^*\}$

d) $L = \{wtw | w, t \in \{0,1\}^*\}$

1.54 Consider the language $F = \{a^i b^j c^k | i, j, k \geq 0 \text{ and if } i = 1, \text{ then } j = k$

a) Show that F is not regular.

• Prove that regular languages are not closed under infinite union.

• Show that the class of regular languages are closed under set difference.

• TRUE or FALSE

1. If $L_1 \cup L_2$ is regular and L_1 is regular, then L_2 is regular.
2. If L_1 is regular and $L_2 \subseteq L_1$, then L_2 is regular.
3. If L_1 is regular and L_2 is not regular, then $L_1 \cup L_2$ is not regular.
4. If L_1 is regular and $L_1 \cup L_2$ is not regular, then L_2 is not regular.
5. If L_1 is regular and L_2 is not regular, then $L_1 \cup L_2$ is not regular.

• Show that union of two non-regular languages is not always non-regular.

PS 4 - 16.03.15

• Midterm Questions

2.4 Give context-free grammars that generate the following languages.

a) $\{w|w$ contains at least three 1's$\}$

b) $\{w|w$ starts and ends with the same symbol$\}$

c) $\{w|$ the length of w is odd$\}$

d) $\{w|$ the length of w is odd and its middle symbol is a $0\}$

e) $\{w|w = w^R\}$

f) The empty set

2.4 Give context-free grammars that generate the following languages.

a) The set of languages over the alphabet $\{a, b\}$ with more a's than b's.

b) The complement of the language $\{a^n b^n | n \geq 0\}$

c) $\{w\#x|w^R$ is a substring of x for $w, x \in \{0, 1\}\}$

2.8 Show that the class of context-free languages are closed under the regular operations union, concatenation and star.

2.10 Give a context-free grammar for the following langauge. $A = \{a^i b^j c^k | i = j$ or $j = k$ where $i, j, k \geq 0\}$. Is your grammar ambigious?

PS 5 - 23.03.15

2.15 Convert the following CFG into Chomsky Normal Form.

$$A \to BAB|B|\epsilon$$
$$B \to 00|\epsilon$$

2.26 Show that if G is a CFG in Chomsky Normal Form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w.

2.5 Give informal descriptions and state diagrams of pushdown automata for the languages in 2.4.

2.44 If A and B are languages, define $A \diamond B = \{xy|x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$. Show that if A and B are regular languages, then $A \diamond B$ is CFL.

• For some $n \geq 1$, does there exist an n-state PDA which accepts finitely many strings, and at least one of those strings is of length n?

• Assume that we modify the PDA model so that the stack now has only a finite capacity. Can this new type of machine recognize any infinite context-free language? Is the set of languages recognized by this new type of machine equal to the set of regular languages?

PS 6 - 30.03.15

• Show that the following language is not context-free: $\{w\#x | w$ is a substring of x for $w, x \in \{0,1\}^*\}$.

2.30 Use the pumping lemma to show that the following languages are not context-free.

a) $\{0^n 1^n 0^n 1^n | n \geq 0\}$

2.31 Let B be the language of all palindromes over $0, 1$ containing an equal number of 0's and 1's. Show that B is not context-free.

• Show that context-free languages are **not** closed under complementation and intersection.

2.18 a) Let C be a context-free language and R be a regular language. Prove that the language $C \cap R$ is context-free.

b) Use part a) to show that the language $A = \{w | w \in \{a, b, c\}^*$ and contains equal number of a's, b's and c's$\}$ is not a CFL.

2.35 Let G be a CFG in Chomsky normal form that contains b variables. Show that if G generates some string with a derivation at least 2^b steps, $L(G)$ is infinite.

3.5 Examine the formal definition of a Turing machine to answer the following questions, and explain your reasoning.

a) Can a Turing machine ever write the blank symbol on its tape?

b) Can the tape alphabet Γ be the same as the input alphabet Σ?

c) Can a Turing machine's head ever be in the same location in two successive steps?

d) Can a Turing machine contain just a single state?

PS 7 - 13.04.15

• Midterm Questions

3.10 A Turing machine with doubly infinite tape is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of Turing machine recognizes the class of Turing-recognizable languages.

3.12 A Turing machine with left reset is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{\text{R, RESET}\}.$$

If $\delta(q, a) = (r, b, \text{RESET})$, when the machine is in state q reading an a, the machine's head jumps to the left-hand end of the tape after it writes b on the tape and enters state r. Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.

3.13 A Turing machine with stay put instead of left is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, S\}.$$

At each point the machine can move its head right or let it stay in the same position. Show that this Turing machine variant is not equivalent to the usual version. What class of languages do these machines recognize?

PS 8 - 27.04.15

3.18 Show that a language is decidable iff some enumerator enumerates the language in lexicographic order.

3.19 Show that every infinite Turing-recognizable language has an infinite decidable subset.

4.10 $\text{INFINITE}_{\text{DFA}} = \{\langle A \rangle |$ is a DFA and $L(A)$ is an infinite language$\}$. Show that $\text{INFINITE}_{\text{DFA}}$ is decidable.

4.13 Let$A = \{< R, S > \mid R$ and S are regular expressions and $L(R) \subseteq L(S)\}$. Show that A is decidable.

4.24 A useless state in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

• If a language L is a Turing recognizable but not decidable, then any TM which recognizes L must fail to halt for infinitely many input strings.

• Let L be the language of all Turing machine descriptions $\langle M \rangle$ such that there exists some input on which M makes at least 5 moves. Show that L is decidable.

PS 9 - 11.05.15

4.20 Let A and B be two disjoint languages. Say that language C separates A and B if $A \subseteq C$ and $B \subseteq \bar{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

4.21 Let $S = \{< M > | M$ is a DFA that accepts w^R whenever it accepts $w\}$. Show that S is decidable.

4.29 Let $C_{\text{CFG}} = \{\langle G, k\rangle | L(G)$ contains exactly k strings where $k > 0$ or $k = \infty\}$. Show that C_{CFG} is decidable.

4.30 Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1), M_2\rangle, \dots\}$, where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A. (Hint: You may find it helpful to consider an enumerator for A.)

• Given an example of a language L such that L is co-Turing recognizable but its complement is not.

• Prove that the language $\{< M, w, q > \mid M$ is a Turing machine which visits state q during its execution when started with input string $w\}$ is undecidable.

• Show that the set of undecidable languages are closed under complementation.

• Prove: A language is Turing recognizable iff there exists an enumerator which enumerates it such that every string in the language appears only once in the listing.

• Disprove: Every countable language is decidable.